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Comparison of the Simulated Phase Sensitivity of Coated and Uncoated Optical Fibers From Plane-Strain Vibration and Static Pressure Models

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PREFACE

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13. ABSTRACT (Maximum 200 words)

A model of the plane-strain vibration of an infinitely long, layered rod with an axisymmetric radial pressure is developed from the exact three-dimensional equations of motion with small argument approximations for the Bessel functions. Applied to both a coated and uncoated optical fiber, this model predicts phase sensitivity using the Pockels coefficients for silica glass. It is shown that the phase sensitivity of the coated optical fiber in forced plane-strain vibration is insensitive to the coating material or coating thickness, while the phase sensitivity of the uncoated optical fiber is equal to the k = 0 results of forced wave propagation in the fiber and is independent of frequency.

A comparison of the phase sensitivity from plane-strain vibration to the phase sensitivity from three static plane-strain models (each with a different boundary condition) is also made. The strains predicted from both models are shown to be identical when a zero axial strain boundary condition is assumed.

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INTRODUCTION

Optical fibers detect a pressure by measuring the pressure-induced change in the phase of an optical signal. In the absence of pressure, the phase of light, ϕ, propagating through a length, L, of optical fiber is

$$\phi = n_o k_o L \; , \tag{1}$$

where n_o is the refractive index of the fiber's core and k_o is the free-space optical wavenumber. A pressure P interacting with the fiber induces a change in phase $\Delta\phi/P$ and is given by[1]

$$\frac{\Delta\phi}{P} = k_o n_o \Delta L + L k_o \Delta n_o \; , \tag{2}$$

where the first term corresponds to the change in the length of the fiber and the second term corresponds to the photoelastic effect. This photoelastic effect describes the relation between the mechanical strain in the fiber and the resulting change in the refractive index.[2]

Generally the *normalized* change in phase is reported by dividing equation (2) by equation (1). If it is assumed that there are no shear strains in the fiber, the normalized change in phase can be expressed as[1]

$$\frac{\Delta\phi}{\phi P} = \varepsilon_{zz} - \frac{n_o^2}{2}\left[(P_{11} + P_{12})\,\varepsilon_{rr} + P_{12}\varepsilon_{zz}\right] \; , \tag{3}$$

where P_{11} and P_{12} are the Pockels coefficients and ε_{rr} and ε_{zz} are the radial and axial strain in the fiber. Substitution of the values of $n_o = 1.45$, $P_{11} = 0.126$, and $P_{12} = 0.270$ into equation (3) results in

$$\frac{\Delta\phi}{\phi P} = 0.712\varepsilon_{zz} - 0.422\varepsilon_{rr} \; . \tag{4}$$

Numerous researchers[1, 3, 4, 5] have used equation (4) to model the optical phase sensitivity of finite-length uncoated and coated optical fibers when they are subjected to a static pressure. In this report, we investigate the case of an infinitely long, coated optical fiber that is subjected to a harmonic, axisymmetric radial pressure. We assume that this pressure is constant along the length of the fiber and that the fiber responds in plane-strain vibration. We compare these results to those from the static pressure models and to the $k = 0$ result of a model of forced wave propagation in an infinitely long, uncoated optical fiber.[6]

PLANE-STRAIN VIBRATION MODEL

SYSTEM DESCRIPTION AND MODELING ASSUMPTIONS

The system geometry, shown in figure 1, consists of an infinitely long, coated rod with a core of radius a, a coating of outer radius b, and an external radial pressure harmonic in time along the axial direction of the form

$$p = Pe^{i\omega t} , \tag{5}$$

where $i = \sqrt{-1}$, ω is frequency (rad/sec), t is time (sec), and P is amplitude (N/m^2). The system displacements and stresses are defined by the cylindrical coordinates, r, θ, and z, which are designated as the 1, 2, and 3 directions, respectively. Assuming that the core and coating are composed of isotropic materials, we can write the displacement equations of motion as[7]

$$\mu_{(m)} u_{i,jj}^{(m)} + (\lambda_{(m)} + \mu_{(m)}) u_{j,ij}^{(m)} = \rho_{(m)} \ddot{u}_i^{(m)} , (m = 1, 2) . \tag{6}$$

In equation (6), λ and μ are the Lame′ constants (N/m^2), ρ is the density (kg/m^3), and u_i are the components of the displacement tensor (m). The superscript and subscript $m = 1$ refers to material properties and displacements of the core, and the superscript and subscript $m = 2$ refers to material properties and displacements of the coating. It should also be noted that Einstein's summation convention applies $(i, j = 1, 2, 3)$, $u_{i,jj}$ is the comma subscripted representation of the second-order partial differentiation of u_i with respect to the variable x_j, $u_{j,ij}$ is the comma subscripted representation of the partial differentiation of u_j with respect to the variables x_j and x_i, and $\ddot{u}_i$ represents the second-order partial differentiation of u_i with respect to time t.

SOLUTIONS TO THE EQUATIONS OF MOTION

We assume plane-strain vibrations so that all displacements and stresses are independent of z. With this assumption, we can write equation (6) as[8]

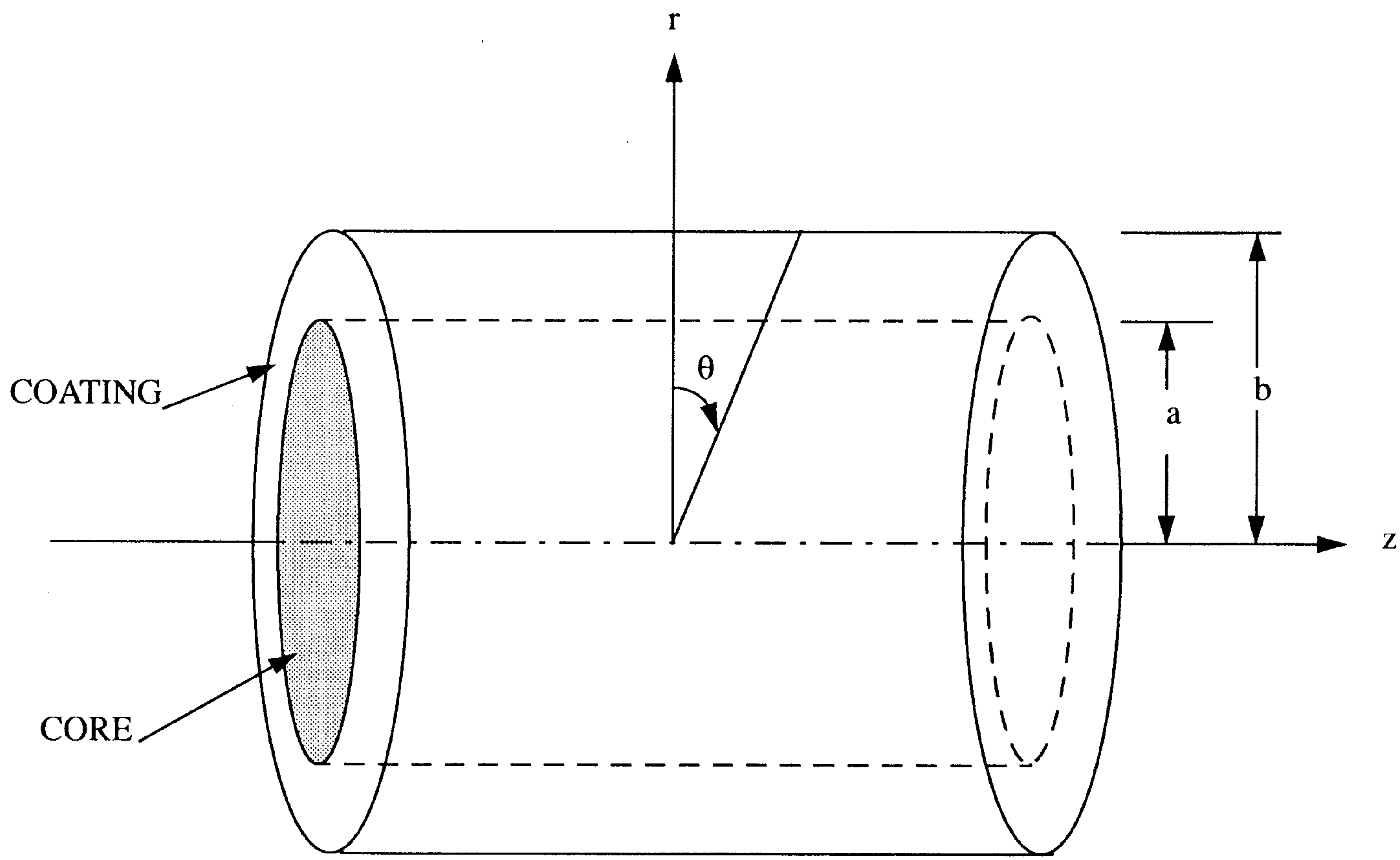

Figure 1.   System Geometry

$$\mu_{(m)}\, u^{(m)}_{\alpha,\beta\beta} + (\lambda_{(m)} + \mu_{(m)})\, u^{(m)}_{\beta,\alpha\beta} = \rho_{(m)}\, \ddot{u}^{(m)}_{\alpha} \,, \tag{7}$$

where $\alpha, \beta = 1, 2$. Gazis has shown[9] that the displacements in the core and in the coating in plane-strain vibration are

$$\begin{aligned} u_r^{(1)} &= A\frac{\partial}{\partial r}[J_0(\Gamma_{(1)} r)]\, e^{i\omega t}, \\ u_z^{(1)} &= 0\,, \end{aligned} \tag{8}$$

and

$$\begin{aligned} u_r^{(2)} &= \{B\frac{\partial}{\partial r}[J_0(\Gamma_{(2)} r)] + C\frac{\partial}{\partial r}[Y_0(\Gamma_{(2)} r)]\}\, e^{i\omega t}, \\ u_z^{(2)} &= 0\,, \end{aligned} \tag{9}$$

respectively, where A, B, and C are constants; J_0 is the Bessel function of the first kind of order zero; and Y_0 is the Bessel function of the second kind of order zero.* The arguments of the Bessel functions in equations (8) and (9) are defined as

$$\Gamma^2_{(1)} = \frac{\rho_{(1)}\,\omega}{\lambda_{(1)} + 2\mu_{(1)}} \tag{10}$$

and

$$\Gamma^2_{(2)} = \frac{\rho_{(2)}\,\omega}{\lambda_{(2)} + 2\mu_{(2)}} \,. \tag{11}$$

The strains and radial stress in the core are (with the time factor $e^{i\omega t}$ omitted here and in the remainder of this section)

$$\begin{aligned} \varepsilon^{(1)}_{rr} &= \frac{\partial u_r^{(1)}}{\partial r}\,, \\ \varepsilon^{(1)}_{\theta\theta} &= \frac{u_r^{(1)}}{r}\,, \end{aligned} \tag{12}$$

and

---

* Actually, Gazis solved the problem of plane-strain vibration for a hollow cylinder and for arbitrary circumferential wavenumber n. We have taken his solution and reduced it to the case of n = 0. We have also modified the solution for a hollow cylinder to that of a solid core by setting the coefficient of the Bessel function of the second kind to zero so that solutions remain finite at the center of the core.

$$\tau_{rr}^{(1)} = (\lambda_{(1)} + 2\mu_{(1)})\,\varepsilon_{rr}^{(1)} + \lambda_{(1)}\varepsilon_{\theta\theta}^{(1)} \ , \tag{13}$$

where ε_{rr} is the radial strain, $\varepsilon_{\theta\theta}$ is the circumferential strain, and τ_{rr} is the radial stress. The strains and radial stress in the coating can be equivalently written by substituting superscripts and subscripts (2) for superscripts and subscripts (1) in equations (12) and (13).

Before proceeding further, we argue that because of the small values of the radius of the core ($a = 62.5 \times 10^{-6}$ meters) and the outer radius of the coating ($a < b < 30\,a$), we may use small argument approximations for the Bessel functions in equations (8) and (9). By using only the first term of the series solution of the Bessel functions[10] in these equations, we find that the displacements, strains, and stresses in the core and coating are simplified to

$$\begin{aligned} u_r^{(1)} &= -\frac{A\Gamma_{(1)}^2 r}{2} \ , \\ u_r^{(2)} &= \frac{-B\Gamma_{(2)}^2 r}{2} + \frac{2C}{\pi r} \ , \end{aligned} \tag{14}$$

$$\begin{aligned} \varepsilon_{rr}^{(1)} &= \frac{-A\Gamma_{(1)}^2}{2} \ , \\ \varepsilon_{\theta\theta}^{(1)} &= \frac{-A\Gamma_{(1)}^2}{2} \ , \\ \varepsilon_{rr}^{(2)} &= \frac{-B\Gamma_{(2)}^2}{2} - \frac{2C}{\pi r^2} \ , \\ \varepsilon_{\theta\theta}^{(2)} &= \frac{-B\Gamma_{(2)}^2}{2} + \frac{2C}{\pi r^2} \ , \end{aligned} \tag{15}$$

and

$$\begin{aligned} \tau_{rr}^{(1)} &= -(\lambda_{(1)} + \mu_{(1)})\,A\Gamma_{(1)}^2 \ , \\ \tau_{rr}^{(2)} &= -(\lambda_{(2)} + \mu_{(2)})\,B\Gamma_{(2)}^2 - \frac{4\mu_{(2)}C}{\pi r^2} \ . \end{aligned} \tag{16}$$

BOUNDARY CONDITIONS

The solutions of the unknown coefficients A, B, and C are found from the boundary conditions of the system. At the solid-solid interface (r = a), the continuity conditions require the radial displacement and stress to be equal. On the outer surface of the coating (r = b), the radial stress is equal to the pressure. These boundary conditions are

$$
\begin{aligned}
u_r^{(1)} - u_r^{(2)} &= 0;\ r = a\,, \\
\tau_{rr}^{(1)} - \tau_{rr}^{(2)} &= 0;\ r = a\,, \\
\tau_{rr}^{(2)} &= -P;\ r = b\,.
\end{aligned}
\tag{17}
$$

Equation (17) represents a system of three linear algebraic equations that in matrix form are

$$
\left[L_{p,q}\right]\{x\} = \{c\} \qquad (p = 1,2,3:\ q = 1,2,3)\,. \tag{18}
$$

The components of the matrixes are defined in appendix A.

It can be shown that (see appendix B) the magnitude of the radial strain in the core of a coated rod in a state of plane-strain vibration, with small argument approximations for the Bessel functions, is independent of frequency.*

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* This assumes that the material properties of both the optical fiber and the coating are independent of frequency.

STATIC PRESSURE MODELS

SYSTEM GEOMETRY

The system geometry for the static pressure models is substantially the same as for the plane-strain vibration model, with the following exceptions:

- The coated rod is assumed to be finite in length.
- The external pressure may also be applied to the ends of the rod.
- The system is assumed to be in a state of plane strain with an average effective axial strain in the core and the coating.

The radial and axial displacements are assumed to be[5]

$$
\begin{aligned}
u_r^{(1)} &= Dr\,, \\
u_z^{(1)} &= w_o z\,, \\
u_r^{(2)} &= Er + \frac{F}{r}\,, \\
u_z^{(2)} &= w_o z\,,
\end{aligned}
\tag{19}
$$

where D, E, F, and w_o are constants. Note that the assumption of an average effective axial strain in the core and the coating results in equal axial displacements. The radial, tangential, and axial strains in the core are

$$
\begin{aligned}
\varepsilon_{rr}^{(1)} &= \frac{\partial u_r^{(1)}}{\partial r} = D\,, \\
\varepsilon_{\theta\theta}^{(1)} &= \frac{u_r^{(1)}}{r} = D\,, \\
\varepsilon_{zz}^{(1)} &= \frac{\partial u_z^{(1)}}{\partial z} = w_o\,,
\end{aligned}
\tag{20}
$$

and the radial, tangential, and axial strains in the coating are

$$\varepsilon_{rr}^{(2)} = \frac{\partial u_r^{(2)}}{\partial r} = E - \frac{F}{r^2}\,,$$

$$\varepsilon_{\theta\theta}^{(2)} = \frac{u_r^{(2)}}{r} = E + \frac{F}{r^2}\,, \tag{21}$$

$$\varepsilon_{zz}^{(2)} = \frac{\partial u_z^{(2)}}{\partial z} = w_o\,.$$

The radial and axial stresses in the core are

$$\begin{aligned}
\tau_{rr}^{(1)} &= \mu_{(1)}\varepsilon_{rr}^{(1)} + \lambda_{(1)}\left(\varepsilon_{\theta\theta}^{(1)} + \varepsilon_{zz}^{(1)}\right) \\
&= 2\,(\lambda_{(1)} + \mu_{(1)})\,D + \lambda_{(1)}\,w_o\,, \\
\tau_{zz}^{(1)} &= (\lambda_{(1)} + \mu_{(1)})\,\varepsilon_{zz}^{(1)} + \lambda_{(1)}\left(\varepsilon_{rr}^{(1)} + \varepsilon_{\theta\theta}^{(1)}\right) \\
&= 2\lambda_{(1)}D + (\lambda_{(1)} + 2\mu_{(1)})\,w_o\,,
\end{aligned} \tag{22}$$

and the radial and axial stresses in the coating are

$$\begin{aligned}
\tau_{rr}^{(2)} &= (\lambda_{(2)} + \mu_{(2)})\,\varepsilon_{rr}^{(2)} + \lambda_{(2)}\left(\varepsilon_{\theta\theta}^{(2)} + \varepsilon_{zz}^{(2)}\right) \\
&= 2\,(\lambda_{(2)} + \mu_{(2)})\,E - \frac{2\mu_{(2)}F}{r^2} + \lambda_{(2)}\,w_o\,, \\
\tau_{zz}^{(2)} &= (\lambda_{(2)} + \mu_{(2)})\,\varepsilon_{zz}^{(2)} + \lambda_{(2)}\left(\varepsilon_{rr}^{(2)} + \varepsilon_{\theta\theta}^{(2)}\right) \\
&= 2\lambda_{(2)}E + (\lambda_{(2)} + 2\mu_{(2)})\,w_o\,.
\end{aligned} \tag{23}$$

BOUNDARY CONDITIONS

In a manner similar to the plane-strain vibration case, continuity conditions require the radial displacements and stresses of the core and coating to be equal at the solid-solid interface and the radial stress on the outer surface of the coating to be equal to the pressure. These boundary conditions are

$$
\begin{aligned}
u_r^{(1)} - u_r^{(2)} &= 0;\ r = a\ , \\
\tau_{rr}^{(1)} - \tau_{rr}^{(2)} &= 0;\ r = a\ , \\
\tau_{rr}^{(2)} &= -P;\ r = b\ .
\end{aligned}
\tag{24}
$$

A fourth boundary condition is required to solve for the four unknown coefficients D, E, F, and w_0. This fourth boundary condition is dependent on the following three assumptions about the axial stress or axial strain in the core and the coating:

$$\sum_m \tau_{zz}^{m} dA_{(m)} = 0\ , \tag{25}$$

$$\sum_m \tau_{zz}^{(m)} dA_{(m)} = -P\pi b^2, \tag{26}$$

and

$$\varepsilon_{zz}^{(1)} = \varepsilon_{zz}^{(2)} = 0\ , \tag{27}$$

where m = 1, 2; A_1 is the cross-sectional area of the core; and A_2 is the cross-sectional area of the coating. Equation (25), which we will call case a, is often referred to as a radial pressure boundary condition with no pressure on the ends. Equation (26), which we will call case b, is often referred to as a hydrostatic boundary condition with pressure on the ends. Equation (27), case c, represents the case of zero axial strain.

Equation (24), along with equation (25), (26), or (27), represents a system of four linear algebraic equations that in matrix form are

$$\left[L_{p,q}\right]\{x\} = \{c\} \qquad (p = 1,2,3,4\text{: } q = 1,2,3,4)\ . \tag{28}$$

The components of the matrixes, for each of the three cases, are defined in appendix C.

Previous researchers[1, 3, 4, 5] have documented the phase sensitivity for coated optical fibers using the assumptions for case a, or case b, or both. Some of this work will be repeated here so that we can compare it to the results for case c.

Case c has not been previously investigated for the phase sensitivity of coated optical fibers. It can be shown (see appendix B) that the results for case c are identical to the results for plane-strain vibration when the small argument approximations are used for the Bessel functions.

SIMULATIONS OF STRAIN COMPONENTS AND PHASE SENSITIVITY

In this section, we will present simulations of the radial and axial strain components and the optical phase sensitivity of coated optical fibers from the static pressure models. These simulations are shown as a function of the ratio of the coating outer radius b to the core radius a (that is, as a function of $R = b/a$) for three coating materials. The three coating materials are Teflon, Hytrel, and Alcryn. Results of investigations of the phase sensitivity of coated fibers in static plane strain (cases a and b) with Teflon and Hytrel coatings have been previously reported.[1,5] In this report we will examine thicker coatings and zero axial strain (case c). The Young's modulus of Alcryn is about two orders of magnitude less than Teflon* and is included in this study to provide the results for coating materials with a broad range of properties.

Figures 2 through 4 compare the radial strain component of phase sensitivity, the axial strain component of phase sensitivity, and the phase sensitivity of case a for Teflon, Hytrel, and Alcryn, respectively. Figures 5 through 7 are similar comparisons of case b. Figure 8 directly compares the phase sensitivity of case c for all three materials. Since the axial strain component has been set equal to zero in case c, the phase sensitivity is equal to the radial strain component.

From figures 2 through 7, we note the following:

- The phase sensitivities of cases a and b are dominated by the axial strain component.
- The axial strain components of cases a and b have significant variation as a function of both the coating thickness and the coating material.
- The radial strain components of all cases have very little variation as a function of coating thickness or coating material, even in case c where we have assumed a zero axial strain component.

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* A summary of the material properties for the three coating materials and for silica glass is contained in appendix D.

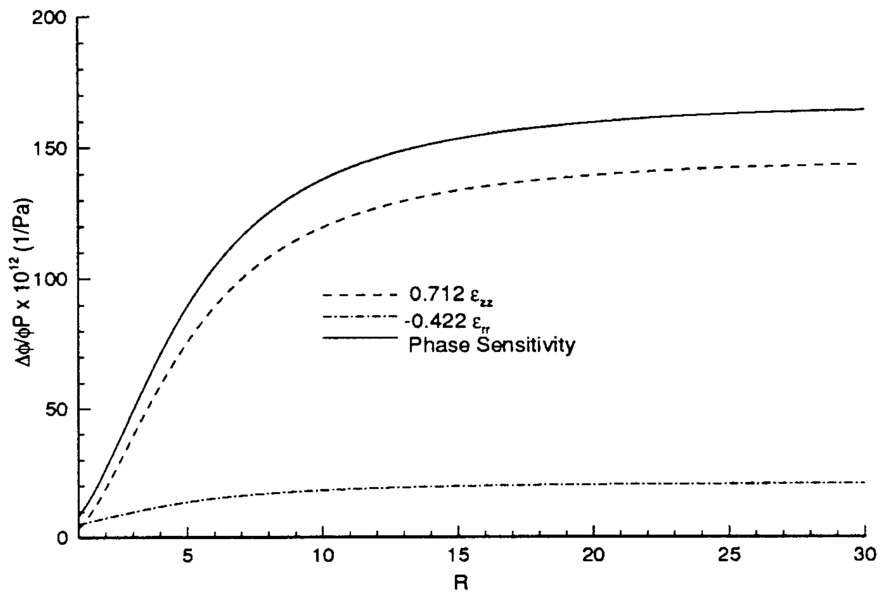

Figure 2. Strain Components and Phase Sensitivity of a Teflon-Coated Optical Fiber in Plane Strain With Radial Boundary Condition (Case a)

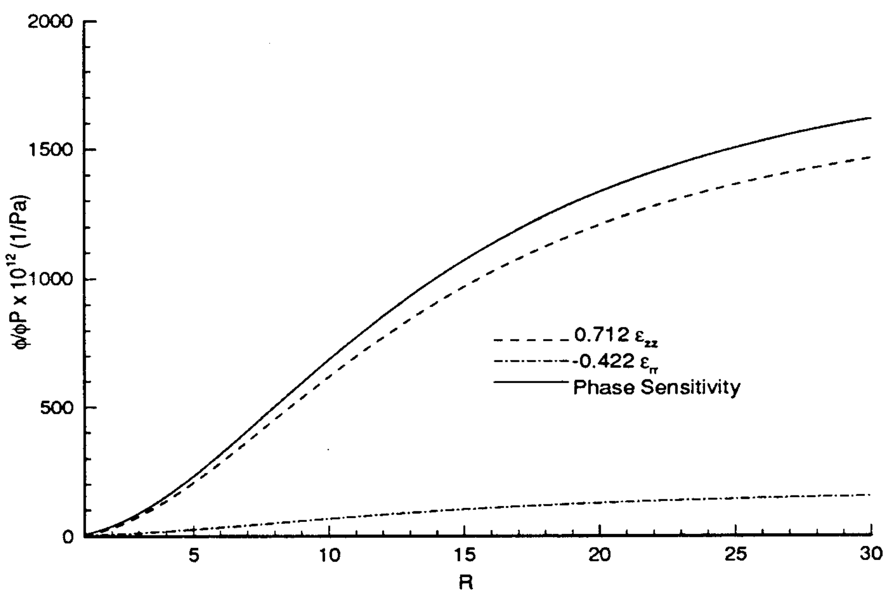

Figure 3. Strain Components and Phase Sensitivity of a Hytrel-Coated Optical Fiber in Plane Strain With Radial Boundary Condition (Case a)

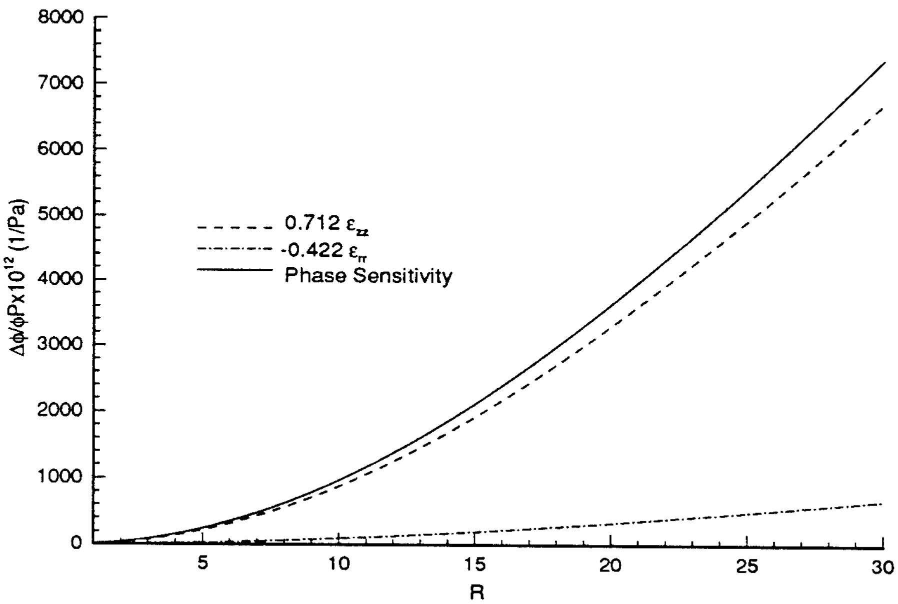

Figure 4. Strain Components and Phase Sensitivity of an Alcryn-Coated Optical Fiber in Plane Strain With Radial Boundary Condition (Case a)

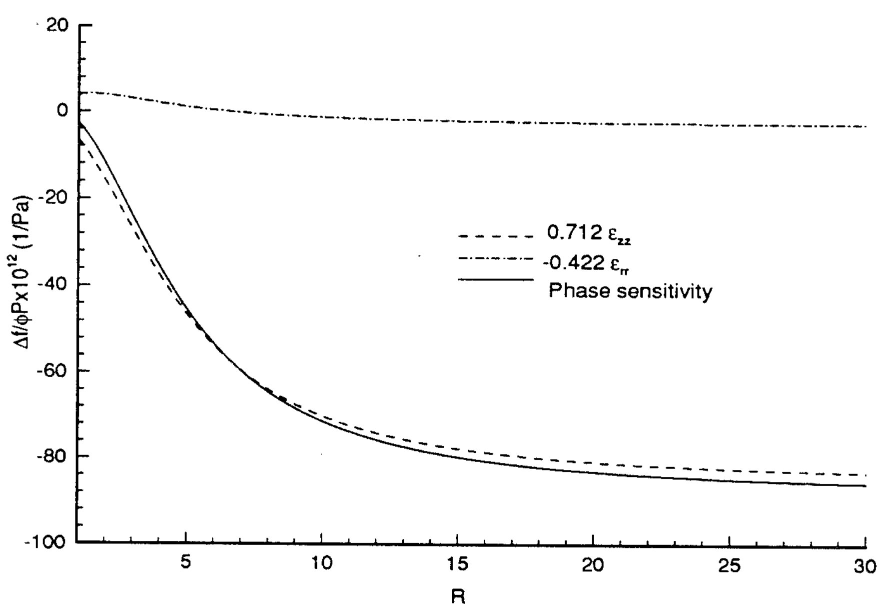

Figure 5. Strain Components and Phase Sensitivity of a Teflon-Coated Optical Fiber in Plane Strain With Hydrostatic Boundary Condition (Case b)

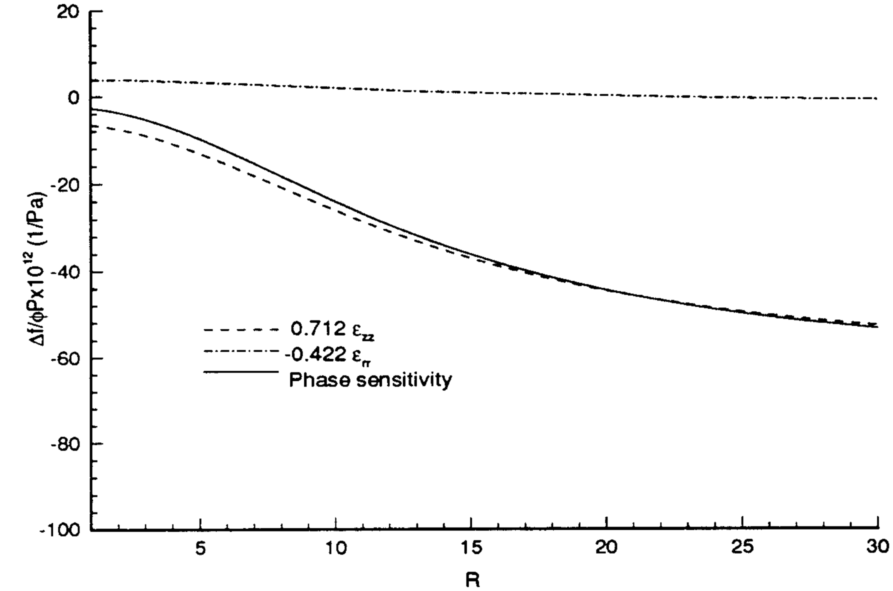

Figure 6. Strain Components and Phase Sensitivity of a Hytrel-Coated Optical Fiber in Plane Strain With Hydrostatic Boundary Condition (Case b)

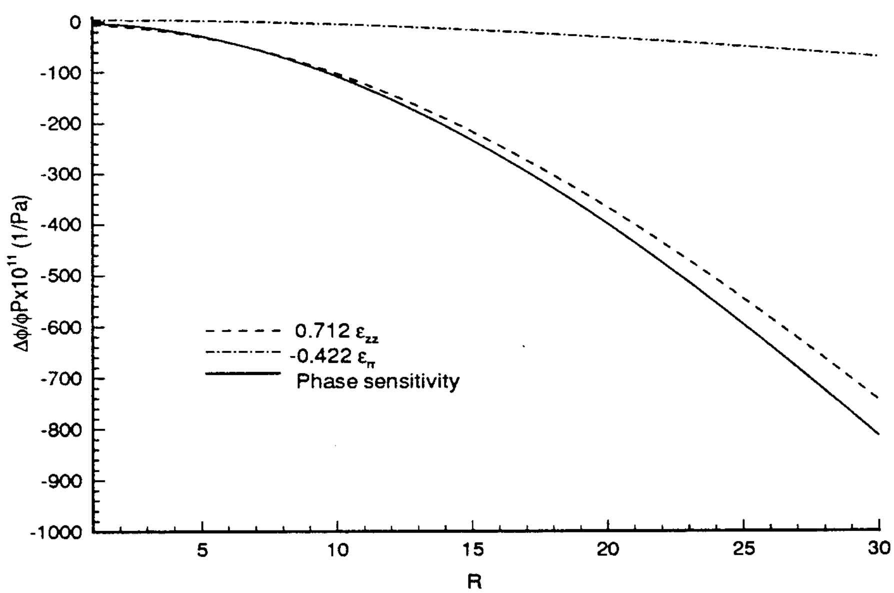

Figure 7. Strain Components and Phase Sensitivity of an Alcryn-Coated Optical Fiber in Plane Strain With Hydrostatic Boundary Condition (Case b)

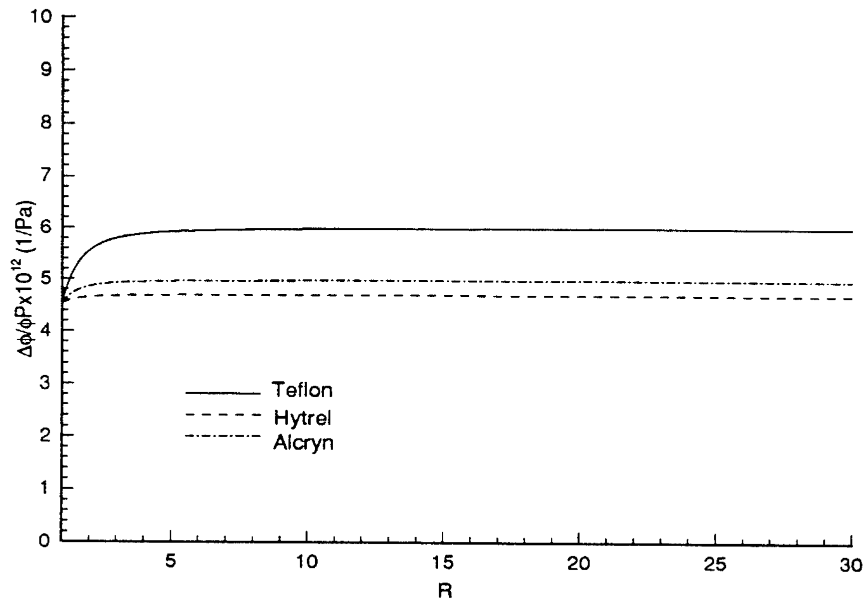

Figure 8. Phase Sensitivity of Teflon-, Hytrel-, and Alcryn-Coated Optical Fibers in Plane Strain With Zero Axial Strain Boundary Condition (Case c)

Table 1 compares the amplification of the phase sensitivity of coated fibers ($\Delta\phi/\phi P$) to the phase sensitivity of uncoated fibers ($\Delta\phi_o/\phi P$) for the three coating materials with $R = 6$ and $R = 30$.

Table 1. Amplification of Phase Sensitivity of Coated Optical Fibers Compared to Uncoated Optical Fibers

Material	$\Delta\phi/\Delta\phi_o$ $R = 6$			$\Delta\phi/\Delta\phi_o$ $R = 30$		
	Case a:	Case b:	Case c:	Case a:	Case b:	Case c:
Teflon	12.68	20.11	1.32	20.01	32.24	1.33
Hytrel	38.52	4.68	1.04	196.48	20.05	1.04
Alcryn	42.86	15.25	1.1	900.81	306.1	1.1

A similar comparison of amplification of phase sensitivity with Teflon and Hytrel as coating materials and with $R = 6$ is contained in reference 5 and is provided here in table 2.

Table 2. Amplification of Phase Sensitivity of Coated Optical Fibers From Reference 5

Material	$\Delta\phi/\Delta\phi_o$ $R = 6$	
	Case a:	Case b:
Teflon	13.0	20.5
Hytrel	40.0	4.8

The minor differences in the values of $\Delta\phi/\Delta\phi_o$ in tables 1 and 2 are due to slight differences in the values of the Young's modulus, Poisson ratio, and Pockels coefficients used in the two analyses. When the values from reference 5* are used in the analysis reported here, the amplification values are identical.

Examination of table 1 shows that the amplification of phase sensitivity is very dependent on R and on the assumed boundary conditions. When $R = 6$, Alcryn provides the most amplification for case a, while Teflon provides the most amplification for cases b and c. However, when $R = 30$, Alcryn provides the most amplification for cases a and b, while Teflon still provides the most amplification for case c. In case c, there is very little difference in the sensitivity amplification, either as a function of the coating material or of R.

Another interesting comparison is that of the $R = 1$ results for case c to the $k = 0$ results of an investigation of forced wave propagation in an uncoated fiber.[6] At $R = 1$, $\Delta\phi/\phi P = 4.526 \times 10^{-12}$ 1/Pa. By taking 10 Log_{10} of the squared magnitude of the phase

---

* Values from reference 5 are contained in appendix D.

sensitivity, we have

$$10\mathrm{Log}\left|\frac{\Delta\phi}{\phi P}\right|^2 = -226.86\ \mathrm{dB}//1/\mathrm{Pa}\ , \tag{29}$$

which is identical to the value in table 1 of reference 6.

CONCLUSIONS AND RECOMMENDATIONS

A static plane-strain model with a zero axial strain boundary condition can be used to predict a plane-strain vibration response when the small argument approximations for the Bessel functions are valid.

When a plane-strain vibration model is used to predict the phase sensitivity of an infinitely long, coated fiber with a uniform, radial harmonic pressure, the results are insensitive to the coating material or the coating thickness.

This report documents a study of the phase sensitivity of coated fibers for the specific case of a uniform ($k = 0$) radial pressure. A more comprehensive model of strain in coated fibers should be developed in order to study the phase sensitivity of coated fibers with a radial harmonic pressure of arbitrary wavenumber and frequency.

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APPENDIX A: PLANE-STRAIN VIBRATION BOUNDARY EQUATIONS

$$L_{1,1} = -\frac{\Gamma_1^2 a}{2} , \tag{A-1}$$

$$L_{1,2} = \frac{\Gamma_2^2 a}{2} , \tag{A-2}$$

$$L_{1,3} = -\frac{2}{\pi a} , \tag{A-3}$$

$$L_{2,1} = -(\lambda_1 + \mu_1)\Gamma_1^2 , \tag{A-4}$$

$$L_{2,2} = (\lambda_2 + \mu_2)\Gamma_2^2 , \tag{A-5}$$

$$L_{2,3} = \frac{4\mu_2}{\pi a^2} , \tag{A-6}$$

$$L_{3,1} = 0 , \tag{A-7}$$

$$L_{3,2} = -(\lambda_2 + \mu_2)\Gamma_2^2 , \tag{A-8}$$

$$L_{3,3} = -\frac{4\mu_2}{\pi b^2} , \tag{A-9}$$

$$\{x\} = \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} , \tag{A-10}$$

and

$$\{c\} = \begin{Bmatrix} 0 \\ 0 \\ -P \end{Bmatrix} . \tag{A-11}$$

APPENDIX B: COMPARISON OF SOLUTIONS

The optical phase sensitivity of a coated fiber, in terms of the axial and radial strains, was given in equation (4) as

$$\frac{\Delta\phi}{\phi P} = 0.712\varepsilon_{zz} - 0.422\varepsilon_{rr} \ . \tag{B-1}$$

For the case of plane-strain vibration and for case c of the static strain models, it was assumed that the axial strain, ε_{zz}, was zero. Therefore, from equation (15), the phase sensitivity in the case of plane-strain vibration becomes

$$\frac{\Delta\phi}{\phi P} = \frac{0.422\frac{A}{P}\Gamma_1^2}{2} \ , \tag{B-2}$$

and from equation (20), the phase sensitivity for case c of the static strain models becomes

$$\frac{\Delta\phi}{\phi P} = -0.422\frac{D}{P} \ . \tag{B-3}$$

Now, solving equation (18) for A and substituting b = Ra, we have

$$\frac{A}{P} = \frac{(\lambda_2 + 2\mu_2)\,R^2}{\xi\Gamma_1^2} \ , \tag{B-4}$$

where

$$\xi = \lambda_1\mu_2 - \lambda_2\mu_2 + \mu_1\mu_2 - \mu_2 + R^2\left(\lambda_1\lambda_2 + \lambda_2\mu_1 + \lambda_1\mu_2 + \lambda_2\mu_2 + \mu_1\mu_2 + \mu_2^2\right) \ . \tag{B-5}$$

When equations (B-4) and (B-5) are substituted into equation (B-2), the phase sensitivity for the case of plane-strain vibration is found to be

$$\frac{\Delta\phi}{\phi P} = \frac{0.422\,(\lambda_2 + 2\mu_2)\,R^2}{2\xi} \ . \tag{B-6}$$

Note that the phase sensitivity in equation (B-6) is independent of frequency. Following a similar procedure, we solve equation (28) for D (with the substitution b = Ra) and find that

$$\frac{D}{P} = \frac{-(\lambda_2 + 2\mu_2)\,R^2}{2\xi} \, . \tag{B-7}$$

When equations (B-5) and (B-7) are substituted into equation (B-3), the phase sensitivity for the case c model of static strain is found to be

$$\frac{\Delta\phi}{\phi P} = \frac{0.422\,(\lambda_2 + 2\mu_2)\,R^2}{2\xi} \, , \tag{B-8}$$

which is identical to equation (B-6).

APPENDIX C: STATIC PRESSURE BOUNDARY EQUATIONS

CASE a:

$$L_{1,1} = a \, , \tag{C-1}$$

$$L_{1,2} = -a \, , \tag{C-2}$$

$$L_{1,3} = -\frac{1}{a} \, , \tag{C-3}$$

$$L_{1,4} = 0 \, , \tag{C-4}$$

$$L_{2,1} = 2\,(\lambda_1 + \mu_1) \, , \tag{C-5}$$

$$L_{2,2} = -2\,(\lambda_2 + \mu_2) \, , \tag{C-6}$$

$$L_{2,3} = \frac{2\mu_2}{a^2} \, , \tag{C-7}$$

$$L_{2,4} = \lambda_1 - \lambda_2 \, , \tag{C-8}$$

$$L_{3,1} = 0 \, , \tag{C-9}$$

$$L_{3,2} = 2\,(\lambda_2 + \mu_2) \, , \tag{C-10}$$

$$L_{3,3} = -\frac{2\mu_2}{b^2} \, , \tag{C-11}$$

$$L_{3,4} = \lambda_2 \, , \tag{C-12}$$

$$L_{4,1} = 2a^2\lambda_1\pi \, , \tag{C-13}$$

$$L_{4,2} = 2\pi\lambda_2\left(b^2 - a^2\right) , \tag{C-14}$$

$$L_{4,3} = 0 \, , \tag{C-15}$$

$$L_{4,4} = \pi\left[a^2\,(\lambda_1 + 2\mu_1) + \left(b^2 - a^2\right)(\lambda_2 + 2\mu_2)\right] , \tag{C-16}$$

$$\{x\} = \begin{Bmatrix} D \\ E \\ F \\ w_o \end{Bmatrix} , \qquad \text{(C-17)}$$

and

$$\{c\} = \begin{Bmatrix} 0 \\ 0 \\ -P \\ 0 \end{Bmatrix} . \qquad \text{(C-18)}$$

CASE b:

(The components of $[L_{p,q}]$ and the components of $\{x\}$ are the same as in case a.)

$$\{c\} = \begin{Bmatrix} 0 \\ 0 \\ -P \\ -P\pi b^2 \end{Bmatrix} . \qquad \text{(C-19)}$$

CASE c:

$$L_{1,1} = a , \qquad \text{(C-20)}$$

$$L_{1,2} = -a , \qquad \text{(C-21)}$$

$$L_{1,3} = -\frac{1}{a} , \qquad \text{(C-22)}$$

$$L_{2,1} = 2(\lambda_1 + \mu_1) , \qquad \text{(C-23)}$$

$$L_{2,2} = -2(\lambda_2 + \mu_2) , \qquad \text{(C-24)}$$

$$L_{2,3} = \frac{2\mu_2}{a^2} , \qquad \text{(C-25)}$$

$$L_{3,1} = 0 , \qquad \text{(C-26)}$$

$$L_{3,2} = 2(\lambda_2 + \mu_2) \ , \tag{C-27}$$

$$L_{3,3} = -\frac{2\mu_2}{b^2} \ , \tag{C-28}$$

$$\{x\} = \begin{Bmatrix} D \\ E \\ F \end{Bmatrix} \ , \tag{C-29}$$

and

$$\{c\} = \begin{Bmatrix} 0 \\ 0 \\ -P \end{Bmatrix} \ . \tag{C-30}$$

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